1. Si $z(x,y) = \ell\text{n}(x^2 + xy + y^2)$, alors

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2.$$

2. Si $z(x,y) = xy + x\exp({}^{y}\!/_{x})$, alors

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z(x,y).$$

3. Si $u(x,y,z) = (x-y)(y-z)(z-x)$, alors

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

4. Si $u(x,y,z) = x + (x-y)/(y-z)$, alors

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 1.$$

5. Avec

$$z(x,y) = \exp(xy) \quad \text{et} \quad u(x) = z\big(x, \varphi(x)\big),$$

on a

$$u'(x) = ye^{xy} + xe^{xy}\varphi'(x).$$

6. On pose

$$z(u,v) = f(x,y) \quad \text{avec} \quad x = uv, \quad y = \frac{u}{v}.$$

Alors

$$\frac{\partial z}{\partial u} = v\frac{\partial f}{\partial x} + \frac{1}{v}\frac{\partial f}{\partial y} \quad \text{et} \quad \frac{\partial z}{\partial v} = u\frac{\partial f}{\partial x} - \frac{u}{v^2}\frac{\partial f}{\partial y}.$$

7. Si $z(x,y) = \varphi(x^2 + y^2)$, alors

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0.$$

8. Exprimer en fonction de f' les dérivées partielles de

$$z(x,y) = f\Big(xy + \frac{y}{x}\Big).$$

9. Si $u = f(x,y,z)$ où $y = \varphi(x)$ et $z = \psi(x,y)$, alors

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\partial f}{\partial x} + \varphi'(x)\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\Big[\frac{\partial \psi}{\partial x} + \varphi'(x)\frac{\partial \psi}{\partial y}\Big].$$

10. Quelle que soit la fonction φ de classe $\mathscr{C}^1$, la fonction

$$z(x,y) = y\varphi(x^2 - y^2)$$

vérifie l'équation

$$\frac{1}{x}\frac{\partial z}{\partial x} + \frac{1}{y}\frac{\partial z}{\partial y} = \frac{z}{y^2}.$$

11. Quelle que soit la fonction φ de classe $\mathscr{C}^1$, la fonction

$$z(x,y) = xy + x\varphi({}^{y}\!/_{x})$$

vérifie l'équation

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z.$$

12. Calculer les dérivées partielles (par rapport à x et à y) des expressions suivantes.

$f_0(x,y) = -x^2y + 2\,xy + y^2 - x$

$f_1(x,y) = \cos\left(-y^2 + 2\,x\right)$

$f_2(x,y) = 6\,x - 3\,y$

$f_3(x,y) = e^{(3\,xy)}$

$f_4(x,y) = e^{\left(-xy^2\right)}$

$f_5(x,y) = x^2e^{(2\,y)}$

$f_6(x,y) = xye^{(2\,x)}$

$f_7(x,y) = x^ye^{\left(-\frac{1}{x}\right)}$

$f_8(x,y) = x^{(y-1)}e^{(-xy)}$

$f_9(x,y) = \cos(2\,xy)$

$f_{10}(x,y) = \cos(x + 2\,y)$

$f_{11}(x,y) = e^{\left(-y^2+2\,x\right)}$

$f_{12}(x,y) = \sin\left(3\,x^2y\right)$

$f_{13}(x,y) = e^{\left(-\frac{x^2}{2\,y}\right)}$

$f_{14}(x,y) = \ell\text{n}\left(\cos\left(x^2y\right)\right)$

$f_{15}(x,y) = \ell\text{n}\left(\cos(x+y) + 1\right)$

$f_{16}(x,y) = \ell\text{n}\left(xe^y\right)$

$f_{17}(x,y) = \ell\text{n}\left(y + e^x\right)$

$f_{18}(x,y) = e^{(\sin(x) - 2\,\sin(y))}$

$f_{19}(x,y) = \sin\left(e^{(2\,x)} - e^{(-y)}\right)$

$f_{20}(x,y) = (2\,x+1)^{3\,y-1}$

$f_{21}(x,y) = \left(y^2+1\right)^{\sin(x)}$

$f_{22}(x,y) = \left(e^{(4\,x-1)}\right)^{y+2}$

$f_{23}(x,y) = \dfrac{2\,x+y}{y^2+1}$

$f_{24}(x,y) = \dfrac{\ell\text{n}(x) - \ell\text{n}(y)}{e^{(y^2)} + e^{(2\,x)}}$

Problème Deux navires appareillent simultanément du point A : l'un se dirige vers le nord à 10 nœuds, l'autre se dirige vers le nord-est à 20 nœuds. Avec quelle vitesse la distance qui les sépare croît-elle ?

$$\frac{\partial f_0}{\partial x}(x,y) = -2\,xy + 2\,y - 1$$
$$\frac{\partial f_0}{\partial y}(x,y) = -x^2 + 2\,x + 2\,y$$

$$\frac{\partial f_1}{\partial x}(x,y) = 2\,\sin\left(y^2 - 2\,x\right)$$
$$\frac{\partial f_1}{\partial y}(x,y) = -2\,y\sin\left(y^2 - 2\,x\right)$$

$$\frac{\partial f_2}{\partial x}(x,y) = 6$$
$$\frac{\partial f_2}{\partial y}(x,y) = -3$$

$$\frac{\partial f_3}{\partial x}(x,y) = 3\,ye^{(3\,xy)}$$
$$\frac{\partial f_3}{\partial y}(x,y) = 3\,xe^{(3\,xy)}$$

$$\frac{\partial f_4}{\partial x}(x,y) = -y^2e^{\left(-xy^2\right)}$$
$$\frac{\partial f_4}{\partial y}(x,y) = -2\,xye^{\left(-xy^2\right)}$$

$$\frac{\partial f_5}{\partial x}(x,y) = 2\,xe^{(2\,y)}$$
$$\frac{\partial f_5}{\partial y}(x,y) = 2\,x^2e^{(2\,y)}$$

$$\frac{\partial f_6}{\partial x}(x,y) = (2\,x+1)ye^{(2\,x)}$$
$$\frac{\partial f_6}{\partial y}(x,y) = xe^{(2\,x)}$$

$$\frac{\partial f_7}{\partial x}(x,y) = (xy+1)x^{(y-2)}e^{\left(-\frac{1}{x}\right)}$$
$$\frac{\partial f_7}{\partial y}(x,y) = x^y e^{\left(-\frac{1}{x}\right)}\,\ell\text{n}\,(x)$$

$$\frac{\partial f_8}{\partial x}(x,y) = -(xy - y + 1)x^{(y-2)}e^{(-xy)}$$
$$\frac{\partial f_8}{\partial y}(x,y) = -(x - \ell\text{n}\,(x))x^{(y-1)}e^{(-xy)}$$

$$\frac{\partial f_9}{\partial x}(x,y) = -2\,y\sin\,(2\,xy)$$
$$\frac{\partial f_9}{\partial y}(x,y) = -2\,x\sin\,(2\,xy)$$

$$\frac{\partial f_{10}}{\partial x}(x,y) = -\sin\,(x+2\,y)$$
$$\frac{\partial f_{10}}{\partial y}(x,y) = -2\,\sin\,(x+2\,y)$$

$$\frac{\partial f_{11}}{\partial x}(x,y) = 2\,e^{\left(-y^2+2\,x\right)}$$
$$\frac{\partial f_{11}}{\partial y}(x,y) = -2\,ye^{\left(-y^2+2\,x\right)}$$

$$\frac{\partial f_{12}}{\partial x}(x,y) = 6\,xy\cos\left(3\,x^2y\right)$$
$$\frac{\partial f_{12}}{\partial y}(x,y) = 3\,x^2\cos\left(3\,x^2y\right)$$

$$\frac{\partial f_{13}}{\partial x}(x,y) = -\frac{xe^{\left(-\frac{x^2}{2\,y}\right)}}{y}$$
$$\frac{\partial f_{13}}{\partial y}(x,y) = \frac{x^2e^{\left(-\frac{x^2}{2\,y}\right)}}{2\,y^2}$$

$$\frac{\partial f_{14}}{\partial x}(x,y) = -\frac{2\,xy\sin\left(x^2y\right)}{\cos\left(x^2y\right)}$$
$$\frac{\partial f_{14}}{\partial y}(x,y) = -\frac{x^2\sin\left(x^2y\right)}{\cos\left(x^2y\right)}$$

$$\frac{\partial f_{15}}{\partial x}(x,y) = -\frac{\sin\,(x+y)}{\cos\,(x+y)+1}$$
$$\frac{\partial f_{15}}{\partial y}(x,y) = -\frac{\sin\,(x+y)}{\cos\,(x+y)+1}$$

$$\frac{\partial f_{16}}{\partial x}(x,y) = \frac{1}{x}$$
$$\frac{\partial f_{16}}{\partial y}(x,y) = 1$$

$$\frac{\partial f_{17}}{\partial x}(x,y) = \frac{e^x}{y+e^x}$$
$$\frac{\partial f_{17}}{\partial y}(x,y) = \frac{1}{y+e^x}$$

$$\frac{\partial f_{18}}{\partial x}(x,y) = e^{(\sin(x)-2\,\sin(y))}\cos\,(x)$$
$$\frac{\partial f_{18}}{\partial y}(x,y) = -2\,e^{(\sin(x)-2\,\sin(y))}\cos\,(y)$$

$$\frac{\partial f_{19}}{\partial x}(x,y) = 2\,e^{(2\,x)}\cos\left(\left(e^{(2\,x+y)}-1\right)e^{(-y)}\right)$$
$$\frac{\partial f_{19}}{\partial y}(x,y) = e^{(-y)}\cos\left(\left(e^{(2\,x+y)}-1\right)e^{(-y)}\right)$$

$$\frac{\partial f_{20}}{\partial x}(x,y) = 2\,(3\,y-1)(2\,x+1)^{(3\,y-2)}$$
$$\frac{\partial f_{20}}{\partial y}(x,y) = 3\,(2\,x+1)^{(3\,y-1)}\,\ell\text{n}\,(2\,x+1)$$

$$\frac{\partial f_{21}}{\partial x}(x,y) = \left(y^2+1\right)^{\sin(x)}\,\ell\text{n}\left(y^2+1\right)\cos\,(x)$$
$$\frac{\partial f_{21}}{\partial y}(x,y) = 2\,y\left(y^2+1\right)^{(\sin(x)-1)}\sin\,(x)$$

$$\frac{\partial f_{22}}{\partial x}(x,y) = 4\,(y+2)e^{(4\,xy+8\,x-y-2)}$$
$$\frac{\partial f_{22}}{\partial y}(x,y) = (4\,x-1)e^{(4\,xy+8\,x-y-2)}$$

$$\frac{\partial f_{23}}{\partial x}(x,y) = \frac{2}{y^2+1}$$
$$\frac{\partial f_{23}}{\partial y}(x,y) = -\frac{4\,xy+y^2-1}{\left(y^2+1\right)^2}$$

$$\frac{\partial f_{24}}{\partial x}(x,y) = \frac{e^{2x}(1+2x\,\ell\text{n}(y/x))+e^{y^2}}{x(e^{y^2}+e^{2x})^2}$$
$$\frac{\partial f_{24}}{\partial y}(x,y) = \frac{e^{y^2}(2y^2\,\ell\text{n}(y/x)-1)-e^{2x}}{y(e^{y^2}+e^{2x})^2}$$